2.1 Exercises

1–4 ■ Express the rule in function notation. (For example, the rule "square, then subtract 5" is expressed as the function $f(x) = x^2 - 5$.)

1. Add 3, then multiply by 2

2. Divide by 7, then subtract 4

3. Subtract 5, then square

4. Take the square root, add 8, then multiply by $\frac{1}{3}$

5–8 ■ Express the function (or rule) in words.

5. $f(x) = \dfrac{x-4}{3}$

6. $g(x) = \dfrac{x}{3} - 4$

7. $h(x) = x^2 + 2$

8. $k(x) = \sqrt{x+2}$

9–10 ■ Draw a machine diagram for the function.

9. $f(x) = \sqrt{x-1}$

10. $f(x) = \dfrac{3}{x-2}$

11–12 ■ Complete the table.

11. $f(x) = 2(x-1)^2$

x	$f(x)$
-1	
0	
1	
2	
3	

12. $g(x) = \lvert 2x + 3 \rvert$

x	$g(x)$
-3	
-2	
0	
1	
3	

13–20 ■ Evaluate the function at the indicated values.

13. $f(x) = 2x + 1$;
$f(1), f(-2), f(\frac{1}{2}), f(a), f(-a), f(a+b)$

14. $f(x) = x^2 + 2x$;
$f(0), f(3), f(-3), f(a), f(-x), f\left(\dfrac{1}{a}\right)$

15. $g(x) = \dfrac{1-x}{1+x}$;
$g(2), g(-2), g(\frac{1}{2}), g(a), g(a-1), g(-1)$

16. $h(t) = t + \dfrac{1}{t}$;
$h(1), h(-1), h(2), h(\frac{1}{2}), h(x), h\left(\dfrac{1}{x}\right)$

17. $f(x) = 2x^2 + 3x - 4$;
$f(0), f(2), f(-2), f(\sqrt{2}), f(x+1), f(-x)$

18. $f(x) = x^3 - 4x^2$;
$f(0), f(1), f(-1), f(\frac{3}{2}), f\left(\dfrac{x}{2}\right), f(x^2)$

19. $f(x) = 2\lvert x - 1 \rvert$;
$f(-2), f(0), f(\frac{1}{2}), f(2), f(x+1), f(x^2+2)$

20. $f(x) = \dfrac{\lvert x \rvert}{x}$;
$f(-2), f(-1), f(0), f(5), f(x^2), f\left(\dfrac{1}{x}\right)$

21–24 ■ Evaluate the piecewise defined function at the indicated values.

21. $f(x) = \begin{cases} x^2 & \text{if } x < 0 \\ x + 1 & \text{if } x \geq 0 \end{cases}$
$f(-2), f(-1), f(0), f(1), f(2)$

22. $f(x) = \begin{cases} 5 & \text{if } x \leq 2 \\ 2x - 3 & \text{if } x > 2 \end{cases}$
$f(-3), f(0), f(2), f(3), f(5)$

23. $f(x) = \begin{cases} x^2 + 2x & \text{if } x \leq -1 \\ x & \text{if } -1 < x \leq 1 \\ -1 & \text{if } x > 1 \end{cases}$
$f(-4), f(-\frac{3}{2}), f(-1), f(0), f(25)$

24. $f(x) = \begin{cases} 3x & \text{if } x < 0 \\ x + 1 & \text{if } 0 \leq x \leq 2 \\ (x-2)^2 & \text{if } x > 2 \end{cases}$
$f(-5), f(0), f(1), f(2), f(5)$

25–28 ■ Use the function to evaluate the indicated expressions and simplify.

25. $f(x) = x^2 + 1$; $f(x+2), f(x) + f(2)$

26. $f(x) = 3x - 1$; $f(2x), 2f(x)$

27. $f(x) = x + 4$; $f(x^2), (f(x))^2$

28. $f(x) = 6x - 18$; $f\left(\dfrac{x}{3}\right), \dfrac{f(x)}{3}$

29–36 ■ Find $f(a)$, $f(a+h)$, and the difference quotient $\dfrac{f(a+h) - f(a)}{h}$, where $h \neq 0$.

29. $f(x) = 3x + 2$

30. $f(x) = x^2 + 1$

31. $f(x) = 5$

32. $f(x) = \dfrac{1}{x + 1}$

33. $f(x) = \dfrac{x}{x + 1}$

34. $f(x) = \dfrac{2x}{x - 1}$

35. $f(x) = 3 - 5x + 4x^2$

36. $f(x) = x^3$

37–58 ■ Find the domain of the function.

37. $f(x) = 2x$

38. $f(x) = x^2 + 1$

39. $f(x) = 2x, \quad -1 \le x \le 5$

40. $f(x) = x^2 + 1, \quad 0 \le x \le 5$

41. $f(x) = \dfrac{1}{x - 3}$

42. $f(x) = \dfrac{1}{3x - 6}$

43. $f(x) = \dfrac{x + 2}{x^2 - 1}$

44. $f(x) = \dfrac{x^4}{x^2 + x - 6}$

45. $f(x) = \sqrt{x - 5}$

46. $f(x) = \sqrt[4]{x + 9}$

47. $f(t) = \sqrt[3]{t - 1}$

48. $g(x) = \sqrt{7 - 3x}$

49. $h(x) = \sqrt{2x - 5}$

50. $G(x) = \sqrt{x^2 - 9}$

51. $g(x) = \dfrac{\sqrt{2 + x}}{3 - x}$

52. $g(x) = \dfrac{\sqrt{x}}{2x^2 + x - 1}$

53. $g(x) = \sqrt[4]{x^2 - 6x}$

54. $g(x) = \sqrt{x^2 - 2x - 8}$

55. $f(x) = \dfrac{3}{\sqrt{x - 4}}$

56. $f(x) = \dfrac{x^2}{\sqrt{6 - x}}$

57. $f(x) = \dfrac{(x + 1)^2}{\sqrt{2x - 1}}$

58. $f(x) = \dfrac{x}{\sqrt[4]{9 - x^2}}$

Applications

59. Production Cost The cost C in dollars of producing x yards of a certain fabric is given by the function

$$C(x) = 1500 + 3x + 0.02x^2 + 0.0001x^3$$

(a) Find $C(10)$ and $C(100)$.

(b) What do your answers in part (a) represent?

(c) Find $C(0)$. (This number represents the *fixed costs*.)

60. Area of a Sphere The surface area S of a sphere is a function of its radius r given by

$$S(r) = 4\pi r^2$$

(a) Find $S(2)$ and $S(3)$.

(b) What do your answers in part (a) represent?

61. How Far Can You See? Due to the curvature of the earth, the maximum distance D that you can see from the top of a tall building or from an airplane at height h is given by the function

$$D(h) = \sqrt{2rh + h^2}$$

where $r = 3960$ mi is the radius of the earth and D and h are measured in miles.

(a) Find $D(0.1)$ and $D(0.2)$.

(b) How far can you see from the observation deck of Toronto's CN Tower, 1135 ft above the ground?

(c) Commercial aircraft fly at an altitude of about 7 mi. How far can the pilot see?

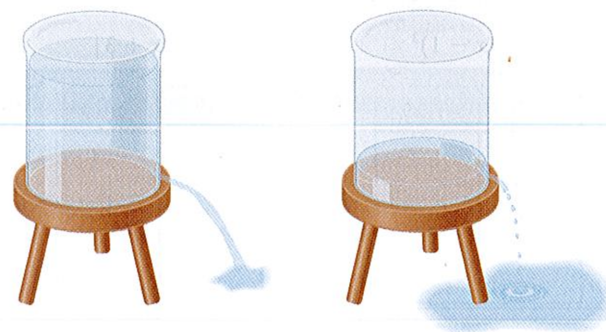
62. Torricelli's Law A tank holds 50 gallons of water, which drains from a leak at the bottom, causing the tank to empty in 20 minutes. The tank drains faster when it is nearly full because the pressure on the leak is greater. **Torricelli's Law** gives the volume of water remaining in the tank after t minutes as

$$V(t) = 50\left(1 - \frac{t}{20}\right)^2 \qquad 0 \le t \le 20$$

(a) Find $V(0)$ and $V(20)$.

(b) What do your answers to part (a) represent?

(c) Make a table of values of $V(t)$ for $t = 0, 5, 10, 15, 20$.



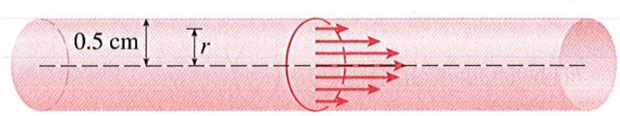
63. Blood Flow As blood moves through a vein or an artery, its velocity v is greatest along the central axis and decreases as the distance r from the central axis increases (see the figure). The formula that gives v as a function of r is called the **law of laminar flow**. For an artery with radius 0.5 cm, we have

$$v(r) = 18{,}500(0.25 - r^2) \qquad 0 \le r \le 0.5$$

(a) Find $v(0.1)$ and $v(0.4)$.

(b) What do your answers to part (a) tell you about the flow of blood in this artery?

(c) Make a table of values of $v(r)$ for $r = 0, 0.1, 0.2, 0.3, 0.4, 0.5$.

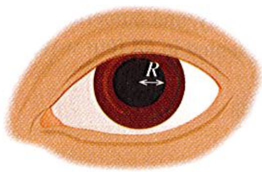


64. Pupil Size When the brightness x of a light source is increased, the eye reacts by decreasing the radius R of the pupil. The dependence of R on x is given by the function

$$R(x) = \sqrt{\frac{13 + 7x^{0.4}}{1 + 4x^{0.4}}}$$

(a) Find $R(1)$, $R(10)$, and $R(100)$.

(b) Make a table of values of $R(x)$.



65. Relativity According to the Theory of Relativity, the length L of an object is a function of its velocity v with respect to an observer. For an object whose length at rest is 10 m, the function is given by

$$L(v) = 10\sqrt{1 - \frac{v^2}{c^2}}$$

where c is the speed of light.

(a) Find $L(0.5c)$, $L(0.75c)$, and $L(0.9c)$.

(b) How does the length of an object change as its velocity increases?

66. Income Tax In a certain country, income tax T is assessed according to the following function of income x:

$$T(x) = \begin{cases} 0 & \text{if } 0 \le x \le 10{,}000 \\ 0.08x & \text{if } 10{,}000 < x \le 20{,}000 \\ 1600 + 0.15x & \text{if } 20{,}000 < x \end{cases}$$

(a) Find $T(5{,}000)$, $T(12{,}000)$, and $T(25{,}000)$.

(b) What do your answers in part (a) represent?

67. Internet Purchases An Internet bookstore charges \$15 shipping for orders under \$100, but provides free shipping for orders of \$100 or more. The cost C of an order is a function of the total price x of the books purchased, given by

$$C(x) = \begin{cases} x + 15 & \text{if } x < 100 \\ x & \text{if } x \ge 100 \end{cases}$$

(a) Find $C(75)$, $C(90)$, $C(100)$, and $C(105)$.

(b) What do your answers in part (a) represent?

68. Cost of a Hotel Stay A hotel chain charges \$75 each night for the first two nights and \$50 for each additional night's stay. The total cost T is a function of the number of nights x that a guest stays.

(a) Complete the expressions in the following piecewise defined function.

$$T(x) = \begin{cases} \qquad & \text{if } 0 \le x \le 2 \\ \qquad & \text{if } x > 2 \end{cases}$$

(b) Find $T(2)$, $T(3)$, and $T(5)$.

(c) What do your answers in part (b) represent?

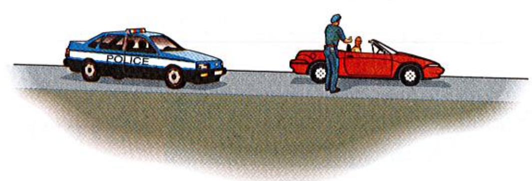
69. Speeding Tickets In a certain state the maximum speed permitted on freeways is 65 mi/h and the minimum is 40. The fine F for violating these limits is \$15 for every mile above the maximum or below the minimum.

(a) Complete the expressions in the following piecewise defined function, where x is the speed at which you are driving.

$$F(x) = \begin{cases} \qquad & \text{if } 0 < x < 40 \\ \qquad & \text{if } 40 \le x \le 65 \\ \qquad & \text{if } x > 65 \end{cases}$$

(b) Find $F(30)$, $F(50)$, and $F(75)$.

(c) What do your answers in part (b) represent?



70. Height of Grass A home owner mows the lawn every Wednesday afternoon. Sketch a rough graph of the height of the grass as a function of time over the course of a four-week period beginning on a Sunday.

71. Temperature Change You place a frozen pie in an oven and bake it for an hour. Then you take it out and let it cool before eating it. Sketch a rough graph of the temperature of the pie as a function of time.

72. Daily Temperature Change Temperature readings T (in °F) were recorded every 2 hours from midnight to noon in Atlanta, Georgia, on March 18, 1996. The time t was measured in hours from midnight. Sketch a rough graph of T as a function of t.

t	T
0	58
2	57
4	53
6	50
8	51
10	57
12	61